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Primordial black holes from cosmic necklaces

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Abstract: Cosmic necklaces are hybrid topological defects consisting of monopoles and strings. We argue that primordial black holes (PBHs) may have formed from loops of the necklaces, if there exist stable winding states, such as coils and cycloops. Unlike the standard scenario of PBH formation from string loops, in which the kinetic energy play important role when strings collapse into black holes, the PBH formation may occur in our scenario after necklaces have dissipated their kinetic energy. Then, the significant difference appears in the production ratio. In the standard scenario, the production ratio f becomes a tiny fraction $f \sim 10^{-20}$, however it becomes $f \sim 1$ in our case. On the other hand, the typical mass of the PBHs is much smaller than the standard scenario, if they are produced in the same epoch. As the two mechanisms may work at the same time, the necklaces may have more than one channel of the gravitational collapse. Although the result obtained in this paper depends on the evolution of the dimensionless parameter r, the existence of the winding state could be a serious problem in some cases. Since the existence of the winding state in brane models is due to the existence of a non-tivial circle in the compactified space, the PBH formation can be used to probe the structure of the compactified space. Black holes produced by this mechanism may have peculiar properties.

Keywords: D-branes, Cosmology of Theories beyond the SM, Black Holes.

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Contents

1. Introduction

Cosmic strings have recently gained a great interest. In the context of a brane world scenario, cosmic strings can be produced after brane inflation [1, 2]. It has been discussed that such strings lead to observational predictions that might be used to distinguish brane world from conventional models [3]. From phenomenological viewpoints, the idea of large extra dimension [4] is important for such models, because it may solve the hierarchy problem. In the scenarios of large extra dimension, the fields in the standard model are localized on wall-like structures, while the graviton propagates in the bulk. In the context of string theory, a natural embedding of this picture is realized by brane construction. The brane models are therefore interesting both from phenomenological and cosmological viewpoints. In order to find signatures of branes, analysis of cosmological defect formation and evolution is important.[1] Brane defects such as monopoles, strings, domain walls and Q-balls are discussed in ref. [11 − 16], where it has been concluded that not only strings but also other defects should appear.

The purpose of this paper is to find distinguishable property of non-standard strings focusing our attention on the production of primordial black holes. S.W. Hawking has discussed [17] that cosmic string loops that shrink by a factor of order $1/G\mu$ will form

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[1] Inflation in models of low fundamental scale are discussed in ref. [5 − 7]. Scenarios of baryogenesis in such models are discussed in ref. [8 − 10], where defects play distinguishable roles.

black holes.[2] He estimated that a fraction f of order $(G\mu)^{2x-4}$ loops will form black holes, where x is a ratio of the loop length to the correlation length. The result obtained in ref. [17] is cosmologically important because the emission of γ-rays from such little black holes could be significant [19]. Numerical simulation of the loop fragmentation and evolution is obtained in ref. [20], where the fraction f becomes

$$f \simeq 10^5 \times (G\mu)^4. \tag{1.1}$$

Black holes created by this collision are so small that they lose their energy due to the Hawking evaporation process. The fraction of the critical density of the Universe in primordial black holes (PBHs) today due to collapsing cosmic string loops is discussed in ref. [21];

$$\Omega_{\mathrm{PBH}}(t_0) = \frac{1}{\rho_{\mathrm{crit}}(t_0)} \int_{t_*}^{t_0} dt \frac{dn_{\mathrm{BH}}}{dt} m(t, t_0). \tag{1.2}$$

Here t_0 is the present age of the Universe, and t_* is the time when PBHs with initial mass $M_* = 4.4 \times 10^{14} g$ are formed, which are expiring today. $m(t, t_0)$ is the present mass of a PBH created at time t, which can be approximated as $m(t, t_0)) \simeq \alpha\mu t$. The extragalactic γ-ray flux observed at 100MeV is commonly known to provide a strong constraint on the population of black holes today. According to ref. [22], the limit implied by the EGRET experiment becomes

$$\Omega_{\mathrm{PBH}} < 10^{-9}. \tag{1.3}$$

The scaling solution of the conventional string network suggests that the rate of PBH formation is

$$\frac{dn_{\mathrm{BH}}}{dt} = f \frac{n_{\mathrm{loop}}}{dt} \sim \alpha^{-1} f t^{-4}. \tag{1.4}$$

Using the above results one can obtain an upper bound [21]

$$G\mu < 10^{-6}, \tag{1.5}$$

which is close to the constraint obtained from the normalization of the cosmic string model to the CMB. Based on the above arguments, we have analyzed in our previous paper the PBH formation in less simplified system [23]. In ref. [23] we have considered two examples; Monopole-antimonopole pairs connected by strings and monopole-string network of the Z_n ($n > 2$) strings. The latter is a model of monopole-string network, in which monopoles are connected to $n > 2$ strings.

Considering the above arguments, it is conceivable that primordial black holes can generally form from both standard string loops and monopole-string networks.[3] In this paper, we consider a more complexified model of monopole-string network; cosmic necklaces.

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[2] See ref. [18] for other possible origin and effects of PBHs.

[3] Here we should note that the PBH production from necklaces is different from the Z_n strings that we have discussed in [23] both in qualitative and quantitative properties. The mechanism that we will consider in this paper is quite different from the scenarios that we have discussed in ref. [23]. In the Hawking's scenario [17] and in ref. [23], the kinetic energy of the collapsing objects (strings or monopoles) play important role. In this case, the gravitational collapse is due to their huge kinetic energy when they collide. On the other hand, the kinetic energy play no role in this paper. This is the crucial difference between our present scenario and the previous scenarios of PBH formation from cosmic defects.

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Berezinsky and Vilenkin [28] found that if one started with a low density of monopoles $r \ll 1$, where r is the ratio of monopole energy density to string energy density per unit length, one can approximate the evolution of the system by the standard evolution of a string network, and also if one could ignore monopole-antimonopole annihilation, the density of monopoles on strings would increase until the point where the conventional-string approximation breaks down. In ref. [28] the authors leave the detailed analysis of the evolution of such systems to numerical simulations, in particular the effect of monopole-antimonopole annihilation. Later in ref. [29] numerical simulations of cosmic necklaces are performed, in which it has been found that the string motion is periodic when the total monopole energy is much smaller than the string energy, and also that the monopoles travel along the string and annihilate with each other. In this paper, we consider networks of necklaces where monopole-antimonopole annihilation is not suppressed. We can therefore approximate the evolution of necklaces by the standard evolution of simple strings, at least during the era when significant amount of primordial black holes are produced.

In the case that the winding states of necklaces are stabilized, the simple statistical argument of a random walk indicates that about $n^{1/2}$ of the initial n monopoles on a long string could survive, even if monopole-antimonopole annihilation is an efficient process.[4] If loops have formed from such strings, heavy winding states (coils) remain [15]. In this case, one can expect that the nucleating rate of such winding states decreases, while the mass increases with time. A similar argument has been discussed in ref. [30] where the authors have assumed that the step length between random walk ($\chi(t)$) is a constant that does not depend on time. On the basis of this assumption, they have obtained the time-dependence of the mass of the winding states (cycloops), $m_{\mathrm{cycloops}} \propto t^{1/2}$. One might think that it should be appropriate to assume that χ "increase" with time due to the expansion of the Universe. However, considering the result obtained in ref. [28] it is obvious that one cannot simply ignore the possibility that χ "decreases" with time. Obviously, this possibility cannot be ignored even in the case that the actual distance between monopoles increases due to annihilation. In this case, efficient annihilation only reduces the number of monopoles from n to $\sqrt{n}$.[5] In this paper, we therefore assume that χ depends on time as

$$\chi(t) \propto t^{k-1}, \tag{1.6}$$

where $k \simeq 0$ corresponds to $\kappa_g - \kappa_s \simeq 1$ in ref. [28]. Since the mass of the winding state always increases with time, one cannot ignore the possibility that the coils could turn into black holes. Of course, Hawking's mechanism of black hole formation from string loops [17] works simultaneously. We will show that in the networks of necklaces-coils the old mechanism of PBH formation is less efficient than the new mechanism.

2. Necklaces stabilized by windings

We consider necklaces whose loops are stabilized by windings around a non-trivial circle in their moduli space. A coil could be a higher-dimensional object (brane) that winds around

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[4] See figure 1.

[5] χ is much smaller than the actual distance between monopoles, $\chi \ll d$.

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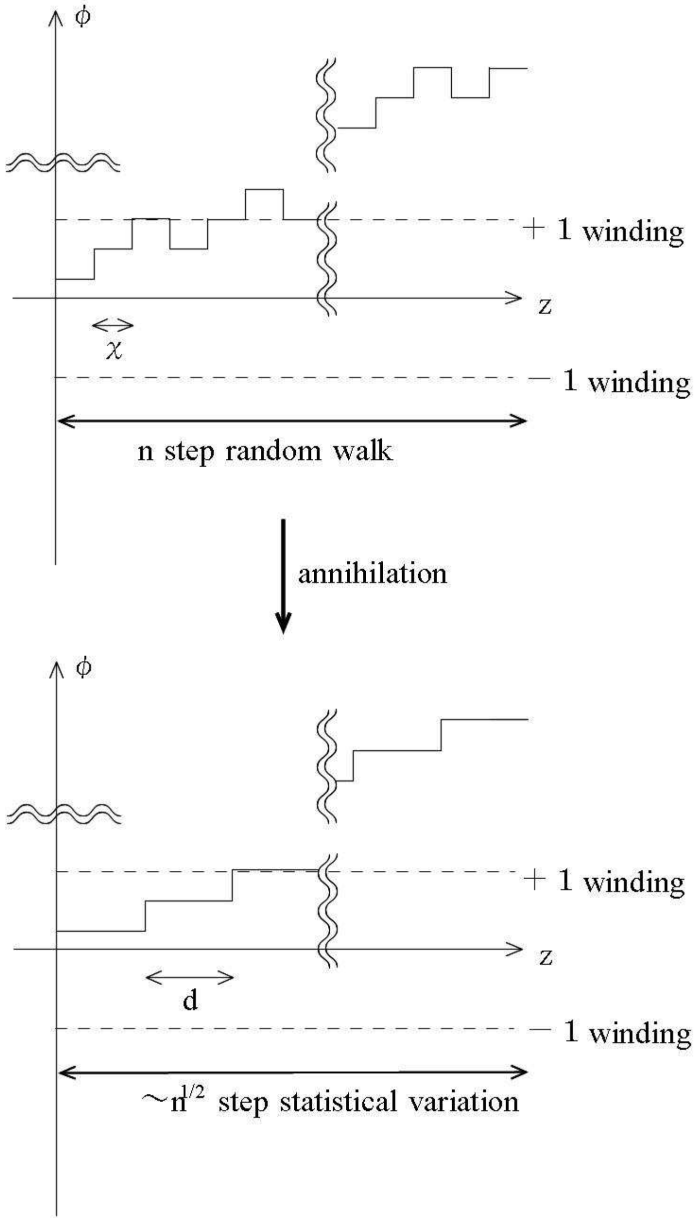

Figure 1: The figure in the first line shows the n-step random walk on a string. Each kink(antikink) corresponds to the left(right) mover. Here the step length between each random walk is denoted by χ. In the case that the annihilation of the kink-antikinks is efficient, about $\sim n^{1/2}$ kinks(antikinks) will remain due to the statistical variation. The distance between kinks that remain after efficient annihilation is denoted by d. In our paper, we assume that the evolution of χ is given by $\chi \propto t^{k-1}$, where the precise value of k could be determined by numerical simulations.

a non-trivial circle in the compactified space, or could be a non-abelian string that has a non-trivial circle in their moduli space. As we have stated above, the simple statistical argument of a random walk indicates that about $n^{1/2}$ of initial n monopoles on a long string could survive when winding states of necklaces are stabilized, even if monopole-antimonopole annihilation is an efficient process. If loops have formed from such strings, heavy winding states (coils) would remain [15]. Here we show two concrete examples. The first is an example of cosmic string produced after brane inflation, and the second is an example of non-abelian necklace.

2.1 Necklaces produced after angled inflation

It is sometimes discussed that only cosmic strings are produced after brane inflation. However, the argument is often based on the assumptions;

- There is only a pair of $D\overline{D}$ brane in the inflation sector.
- There is no "overproduction" [16]
- There are no local vacua in the moduli space of such strings so that no kinks appear on the strings. One should also assume that strings cannot move and wind around a non-trivial circle in the compactified space.

However, as we have discussed in ref. [11, 12, 15], defects other than strings can be produced in generic models of brane inflation. Strings turn into necklaces [15] when there exists (quasi-)degenerated local vacua in their moduli space. In this case, kinks appear on strings interpolating between domains of (quasi-)degenerated vacua. One can also consider windings around internal space, which is called "coils" in ref. [15] and later named "cycloops" in ref. [30]. Perhaps the simplest scenario of $D\overline{D}$ annihilation is rather problematic if one take seriously the mechanism of reheating [32]. It is therefore important [14, 15] to consider less simplified model of brane inflation. For example, angled inflation is a model of brane inflation in which inflating branes does not merely annihilate but reconnect at the end of inflation. In this case the daughter branes are not free propagating in the bulk, but have their endpoints on their mother branes. To make our discussions simple and convincing, here we consider angled brane inflation with a small angle ($\theta \ll 1$) [14, 15]. In angled inflation, cosmic strings are the daughter D_{p-2} branes extended between mother D_p branes. The D_{p-2} branes have the flat direction (moduli) in the compactified space, along which the endpoints of the D_{p-2} branes can move freely on the D_p branes. Then the position of the D_{p-2} branes can vary along a cosmic string, which results in $(1+1)$-dimensional kink configurations appearing on the string. One can also find a winding state that is defined on the worldvolume of the strings [15]. In this case, kinks that appear on strings are monopoles, and they are produced by spatial deformation of the D_{p-2} branes. Therefore, the brane necklaces are the *hybrid* of "brane creation" and "brane deformation". The chopped loops of the necklaces can shrink to produce stable winding states, which look like coils winding around compactified space. It will be helpful to note here that a brane coil winds around the compactified space that is *different* from the mother branes. Of course the mechanism that induces such windings is different from the conventional Kibble mechanism, therefore our argument does not contradict to the previous arguments [33] that have suggested only strings are produced after brane inflation.

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2.2 Non-abelian necklaces

Besides the brane-induced necklaces that we have discussed above, one can construct another model that have a similar configuration without invoking brane dynamics in compactified space. In this case, the flat direction in the compactified space is replaced by a moduli space of the worldvolume effective action of a non-abelian string. Let us consider the dynamics of cosmic strings living in a non-abelian $\mathrm{U}(N_c)$ gauge theory that is coupled to N_f scalar fields q_i, which transform in the fundamental representation;

$$L = \frac{1}{4e^2} Tr F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} \mathcal{D}_\mu q_i^\dagger \mathcal{D}_\mu q_i^\dagger - \frac{\lambda e^2}{2} \left(\sum_{i=1}^{N_f} q_i \otimes q_i^\dagger - v^2 \right)^2 . \tag{2.1}$$

The above Lagrangian has a $\mathrm{SU}(N_f)$ flavor symmetry, which rotates the scalars. We can also include explicit symmetry breaking terms into the Lagrangian, which breaks global flavor symmetry. The most obvious example is a small mass term for the scalars;

$$V_{br1} \sim \sum_i m_i^2 q_i^\dagger q_i, \tag{2.2}$$

which shifts the vacuum expectation value to [34]

$$q_i^a = \left(v^2 - \frac{m_i^2}{\lambda e^2} \right)^{1/2} \delta_i^a . \tag{2.3}$$

Then one may embed an abelian vortex in the i-th U(1) subgroup of $\mathrm{U}(N_c)$, whose tension becomes $T_i \sim \left(v^2 - \frac{m_i^2}{\lambda e^2} \right)^{1/2}$. In this case, due to the difference between the string tension, monopoles could form binding states.

One may extend the model to $N = 2$ supersymmetric QCD or simply include an additional adjoint scalar field ϕ. In any case, the typical potential for the adjoint scalar could be [34]

$$V_{br2} \sim \sum_i q_i^\dagger (\phi - m_i)^2 q_i . \tag{2.4}$$

Here the potential breaks $\mathrm{U}(1)_R$ symmetry. The tensions of the strings degenerate in this case.

Alternatively, one can consider supersymmetry-breaking potential that could be induced by higher-dimensional effects,

$$V_{br3} \sim \sum_i q_i^\dagger (|\phi|^2 - m^2) q_i , \tag{2.5}$$

which preserves $\mathrm{U}(1)_R$ symmetry. In this case, due to D-flatness condition if supersymmetry is imposed, the vacuum expectation value of the adjoint field is placed on a circle and given by [12]

$$\phi = m \times \mathrm{diag} \left(1, e^{\frac{2\pi}{N_c}}, e^{\frac{2\pi}{N_c} \times 2}, \ldots e^{\frac{2\pi}{N_c} \times (N_c - 1)} \right) . \tag{2.6}$$

One can break the remaining classical $\mathrm{U}(1)_R$ symmetry by adding an explicit breaking term, or by anomaly [12, 34].

In any case, strings living in different U(1) subgroups can transmute each other by the kinks (walls on their 2D worldvolume) that interpolate between degenerated vacua.

Is it possible to construct "winding" state from non-abelian necklaces, which look like coils in brane models? A similar argument has already been discussed by Dvali, Tavartkiladze and Nanobashvili [35] for Z_2 domain wall in four-dimensional theory. The authors have discussed that "windings" may stabilize the wall-antiwall bound state if the potential is steep in the radial direction. Of course, a similar situation may happen if (for example) the origin is lifted by an effective potential $\sim \phi^{-n}$. The important point here is whether the absolute value of the scalar field vanishes inside the bound state of walls (kinks). If "windings" of such kinks cannot be resolved due to the potential barrier near the origin, naive annihilation process is inhibited and stable bound state will remain. The same "windings" may happen in our case, where kinks are corresponding to domain walls on (1+1)-dimensional world. The winding states are therefore generic remnants of non-abelian necklaces.

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2.3 Distance between monopoles

In this paper, we assume that monopole-antimonopole annihilation is efficient so that $r \ll 1$ is a good approximation. Since we are considering a model in which the winding states of necklaces are stabilized, the simple statistical argument of a random walk indicates that about $n^{1/2}$ of the initial n monopoles on a long string could survive even if monopole-antimonopole annihilation is an efficient process. If loops have formed from such strings, stable winding states would remain. In any case, the initial distance between monopoles is important. Typically the number density to entropy ratio is expressed by the standard formula

$$\frac{n_m}{s} > \left[\left(\frac{T_M}{M_p}\right) ln \left(\frac{M_p^4}{T_M^4}\right)\right]^3. \tag{2.7}$$

Therefore, distance between monopoles when necklaces are formed at the temperature $T = T_n$(or at the time $t = t_n$) is bounded by [25]

$$d(t_n) < (t_n t_M)^{1/2} \sim T_n^{-1}\left(\frac{M_p}{T_M}\right), \tag{2.8}$$

which gives us the *maximum* value of χ at $t = t_n$.[6]

2.4 Mass of coils

Let us assume that a loop of the length $l(t)$ initially contains n monopoles. Then from the simple statistical argument of a random walk one can obtain the mass of the stable relic;

$$M_{\rm coil}(t) \sim n(t)^{1/2} m. \tag{2.9}$$

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[6] In brane models, if monopole production occurs later than the string production, what appears during the period between string production and t_n is called cycloops [30]. After t_n, when the lift of the potential becomes significant, cycloops turns into necklaces on which kinks (beads) are interpolating between vacua.

Here we assume that the number of monopoles that are initially contained in a loop is given by

$$n(t) \sim \frac{l(t)}{d(t_n) \times \left(\frac{t}{t_n}\right)^{k-1}}. \tag{2.10}$$

For example, in the case when $k = 0$, the initial number of monopoles that could be contained in a loop that is just chopped off from the string networks is

$$n(t) \sim \frac{l(t)}{d(t_n) \times \left(\frac{t}{t_n}\right)^{-1}}. \tag{2.11}$$

It will be helpful to comment about the crucial points related to the time-dependence of the mass of the winding states.

1. In the standard scenario [17], it has been discussed that the mass of PBHs increases with time as $m_{\rm PBH} \sim \alpha\mu t$, while the production rate $f \sim 10^{-20}$ is a tiny fraction. In our case, the mass of PBHs depends on time as

   $$m_{\rm PBH} \propto t^{\frac{2-k}{2}} \tag{2.12}$$

   where k may have gap at $t = t_{\rm eq}$. In our case, the mass of PBHs could be either small($k < 0$) or large($k > 0$) compared to the original scenario. The bound in our model is therefore quite sensitive to the value of k, which suggests that numerical simulations are quite important for further study.

2. The present number density of black holes with mass M can be obtained by redshifting the distribution from the time of their formation to the present time t_0. Then we can calculate $dn_{\rm BH}/dM$. For example, let us consider $k = \delta$ when there is a small ($\delta \ll 1$) deviation from $k = 0$. Then, we have

   $$\frac{dn_{\rm BH}(M)}{dM} \propto M^{-2.5-0.75\delta}. \tag{2.13}$$

   Obviously, deviation from the standard mass-dependence is a unique property of our scenario. This can be used to distinguish conventional strings from necklaces/coils.

2.5 Loops from scaling necklaces

Let us assume that PBH formation starts after friction-domination has been ended at $t = t_{\rm scale}$. In this case, one can assume that the typical velocity of necklaces is $v \sim 1$. If the monopoles does not have any unconfined magnetic charge, one can assume that necklaces scale and the number of loops produced from the network is

$$\frac{dn(t)_l}{dt} \sim \frac{1}{\alpha N^{-1} t^4}, \tag{2.14}$$

where N is the number of (quasi)-degenerated vacua of the necklace. Our result is based on the assumption that the evolution of the network of the necklaces is identical to the strings of low reconnection ratio; $p \sim N^{-1} \ll 1$ [15].

2.6 PBHs from necklaces

As we have discussed above, mass of the coils tends to increase with time. In our case, although the typical length of loops increases as $l \propto t$, the mass of the coils depends on time as $m_{\rm coil} \propto t^{\frac{2-k}{2}}$. Initially the motion of necklaces is damped due to frictional force until $t_{\rm scale} \sim (G\mu)^{-1} t_s$, where t_s is the time of string formation. After damped epoch ($t > t_{\rm scale}$), network of necklaces starts scaling.[7] Although it might depend on the form of the potential that induces kinks on the strings, k_n kinks (monopoles) on long string will have width $\delta_k > k_n \times \delta_m$, where δ_m denotes the width of a monopole. One can therefore assume that monopoles cannot turn into PBHs as far as they are moving on a long string. A chopped string form loops that can shrink to a point (at least in the uncompactified space) after they have dissipated their kinetic energy.[8] Even in the case that the width of a kink is wide due to a shallow potential in the moduli space, these kinks can turn into a point-like winding state. Here we do not discuss about the details of the condition, but simply assume that we are considering a model in which loops can shrink to a point in the four-dimensional space due to their string tension.[9]

Now we can estimate the time when coils start to turn into black holes. The Schwarzschild radius of a loop which contained initially n monopoles is

$$R_g \sim \sqrt{n(t)}m/M_p^2, \tag{2.15}$$

where m is the mass of a monopole. In the case when a loop shrinks to a point-like state whose width is $\delta_{\rm coil} \sim \eta_s^{-1}$, the condition for PBH formation becomes

$$R_g > \eta_s^{-1}. \tag{2.16}$$

Using eq. (2.10), (2.15) and (2.16), we can calculate $t_{\rm PBH}$ when PBH formation starts;

$$t_{\rm PBH} \sim \left(\frac{M_p^4 d(t_n) t_n^{1-k}}{\eta_s^2 m^2 \alpha} \right)^{1/(2-k)} . \tag{2.17}$$

To calculate the present density of primordial black holes, we should consider black holes whose mass is larger than M_*, as we have discussed in section 1. In the natural evolution ($k = 0$), one can easily obtain a severe bound for the string tension [21]. In this case, the explicit form of the mass is

$$m_{\rm coil} \sim \alpha^{1/2} (d(t_n) t_n)^{-1/2} m t. \tag{2.18}$$

From eq. (2.14), one can obtain

$$G\mu < 10^{-20} \times \left[\frac{p}{10^{-2}}\right]^{4/5} \left[\frac{\gamma}{10^2}\right]^{1/5} \left[\frac{t_n}{M_p/\mu}\right]^{3/5} \left[\frac{d(t_n)}{M_p/\mu}\right]^{3/5} \left[\frac{m}{10^{16} GeV}\right]^{-6/5}, \tag{2.19}$$

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[7] Although the constant k could have a gap at $t = t_{\rm scale}$, here we assume that the gap is negligible.

[8] If one wants to discuss PBHs from loops *before* dissipation, the ratio becomes tiny [17]. Note that what we are considering here is different from the original scenario [17].

[9] See figure 2.

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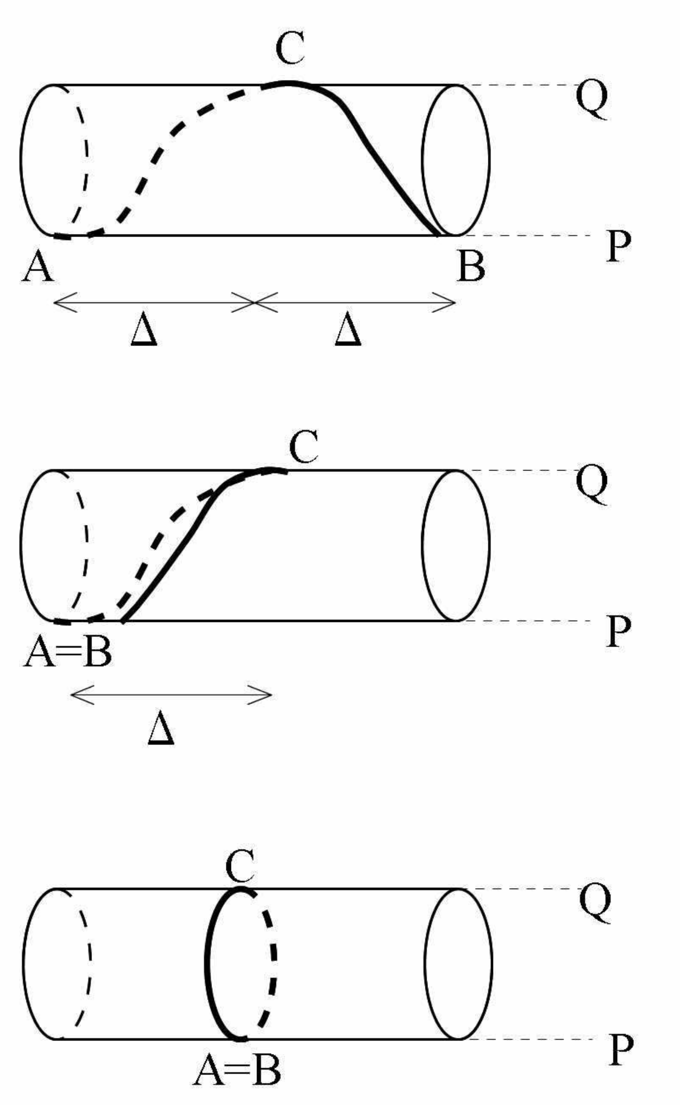

Figure 2: The figure in the first line shows the winding that appears on a long necklace. Here the width of the kinks $A - C$ and $C - B$ is denoted by Δ. There are degenerated vacua at A, B and C that appear on the necklaces. The figure in the middle shows a loop that is formed by the necklace. In this case, the left end is identified with the right. The figure in the last line shows the conventional winding state of the brane. In general, the tension of the strings are much larger than the square of the effective mass that lifts the flat directions. Then, the width of the winding state becomes about the same order as the width of the brane itself, as is shown in the last line. As we are not considering any peculiar repulsive force acting between the D-branes, the width of the n-winding state cannot be proportional to n.

where we have assumed $\alpha \sim \gamma G\mu$ and followed the argument in ref. [21]. In this case, although the "initial" number density of monopoles that could be contained in a loop increases with time, one may obtain $r \ll 1$ if annihilation is an efficient process. To be precise, assuming efficient annihilation, the actual number density of monopole per unit

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length may become a constant (i.e. $r = \text{const.}$), which suggests that the ratio r have an attractor point at $k = 0$.[10]

3. PBHs from cycloops

Here we follow the argument presented in ref. [30]. According to ref. [30], here we assume that there was a period when strings can move freely in extra dimensions. The number of black holes created at time t is given by eq. (2.14), and the mass of black holes produced by a chopped loop at t is

$$m(t) \sim \omega_l \mu \alpha t, \tag{3.1}$$

where ω_l is a parameter that was used in ref. [30]. For a loop of total length $l_{\rm cy}$, its length in the compact dimensions is given by $l_{\rm cy}\omega_l$. It should be noted that before the "lift" of the potential one can use the result that is obtained in ref. [30] for velocity correlations. After the "lift", one can use the result obtained in our paper. It is natural to think that the production of PBH starts much later than the "lift" of the potential. Then it is easy to obtain an upper bound [21];

$$G\mu < 10^{-15} \times \left[\frac{p}{10^{-2}}\right]^{1/2} \left[\frac{\gamma}{10^{2}}\right]^{-1/4} \omega_l^{-3/4} \tag{3.2}$$

where we have assumed $\alpha \sim \gamma G\mu$.

4. Conclusions and discussions

The effective action of the daughter branes that are produced by brane collision could have flat directions corresponding to their free motion in the compactified space. In this case, if the compactified manifold is not simply connected, or valley of the potential for the string motion in extra dimensions has a non-trivial circle, stable winding states may appear. Moreover, the effective potential that lifts the flat directions may have degenerated vacua. Then, due to the random distribution of the "vacua", cosmic strings will turn into necklaces and coils. In this paper, we have considered PBH formation from loops of cosmic necklaces. The existence of the winding states could be a serious problem. Since the existence of the winding state in brane models is due to the existence of a non-tivial circle in the compactified space, the PBH formation can be used to probe the structure of the compactified space. Black holes produced by this mechanism may have peculiar properties that can be used to probe extra dimensions.

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[10] The possibility that r have an attractor is suggested in [28].

A. From cycloops to necklaces-coils

Here we make more comments about the scenario of cycloops [30], which should be helpful to understand why necklaces-coils are more important in our analysis. In ref. [30], cosmic loop production by long string interactions in models with compact extra dimensions was studied, where the important assumption is free motion in the extra dimensions. In the case that the compact manifold is not simply connected, there is a possibility that loops wrap around non-trivial circles. The authors claimed that cycloops poses a potential monopole problem because such loops behave like heavy matter in radiation era, and calculated the energy density of cycloops. Then they have shown that to avoid cycloop domination the strings must satisfy the severe constraint $G\mu < 10^{-14}$, which may be consistent with brane inflation. In this scenario, however, the authors claimed that the mass of cycloops increase with time as $m_{\rm cycloops} \propto t^{1/2}$ if the strings obey statistical model of random walk, and also that $m_{\rm cycloops} \propto t$ if velocity correlations are considered. Therefore, one can easily understand that the PBH formation must be crucial in the scenario of cycloops, because in this case the production ratio of PBHs from loops cannot be suppressed, while the mass increases with time as $m_{\rm cycloops} \propto t$. Here we should note that, even in the standard scenario of PBH formation where the PBH production ratio is highly suppressed as $f < 10^{-20}$, the upper bound obtained from PBH constraint is as low as $G\mu < 10^{-6}$. Moreover, the present fraction of PBHs must satisfy (1.3), which is much stronger than $\Omega_{\rm cycloops} < 1$ which has been considered in ref. [30]. It is therefore important to understand PBH formation in the scenario of cycloops.

However, since PBH production starts at late epoch, one cannot assume free motion in the extra dimensions, but should consider the "lift" from the potential.[11] This is the reason why one should consider necklaces/coils rather than cycloops.[12] Here we should note that one cannot simply ignore the effective potential that lifts the moduli, particularly the one lifts the flat direction that corresponds to the string motion in the internal space. If the potential is so high that the strings cannot move in the extra dimensions, necklaces/coils are unlikely. However, if strings can climb up the potential hill at least in the most energetic epoch when strings are just produced, there could be random distribution of "vacua" on the strings and kinks that interpolate between them.

If the potential stabilizes the vacuum, one should consider cosmic necklaces/coils instead of cycloops. Therefore, here we consider the network of necklaces/coils in scaling epoch, because the production of primordial black holes starts late.

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[11] The situation is shown in figure 3.

[12] We will discuss dark matter production in our forthcoming paper [31].

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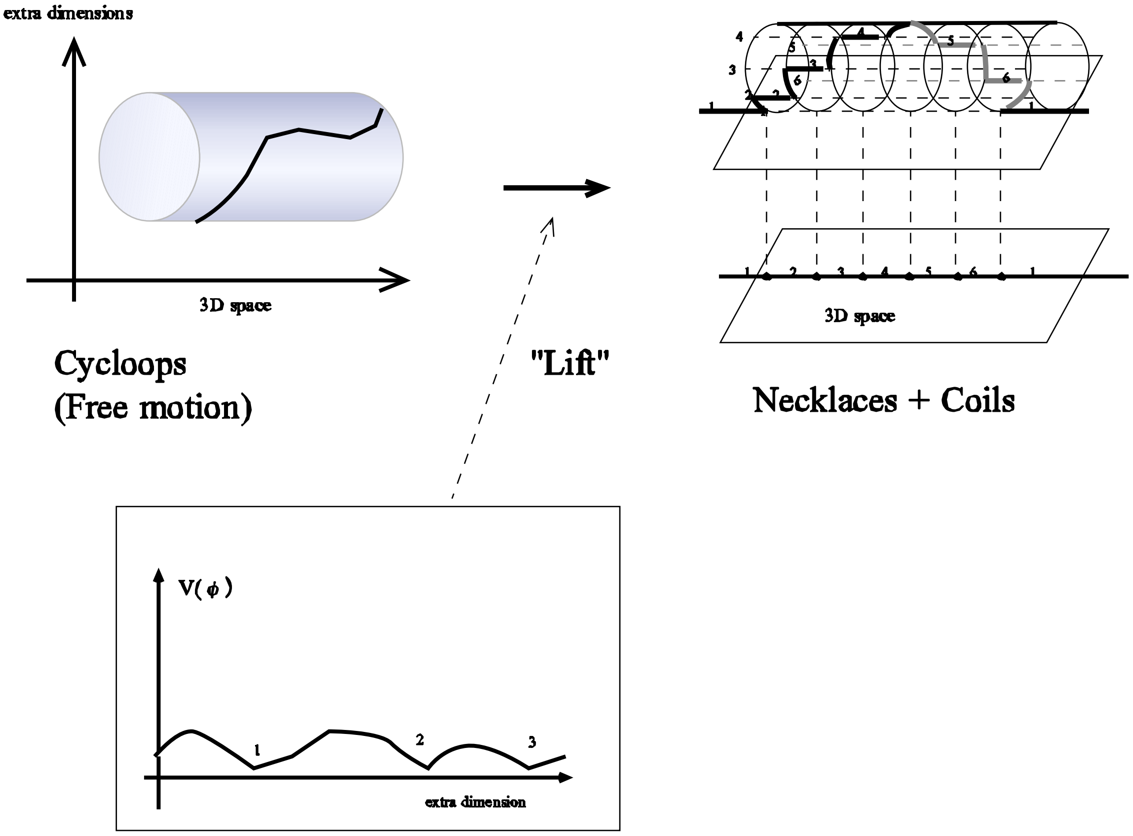

Figure 3: This picture shows the relation between cycloops and necklaces-coils.

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